Determination of Markov Transition Probabilities to be Used in Bridge Management from Mechanistic-Empirical Models

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Abstract: Many bridge management systems use Markov models to predict the future deterioration of structural elements. This information is subsequently used in the determination of optimal intervention strategies and intervention programs. The input for these Markov models often consists of the condition states of the elements and how they have changed over time. This input is used to estimate the probabilities of transition of an object from each possible condition state to each other possible condition state in one time period. A complication in using Markov models is that there are situations in which there is an inadequate amount of data to estimate the transition probabilities using traditional methods (e.g., due to the lack of recording past information so that it can be easily retrieved, or because it has been collected in an inconsistent or biased manner). In this paper, a methodology to estimate the transition probabilities is presented that uses proportional data obtained by mechanistic-empirical models of the deterioration process. A restricted least-squares optimization model is used to estimate the transition probabilities. The methodology is demonstrated by using it to estimate the transition probabilities for a reinforced concrete (RC) bridge element exposed to chloride-induced corrosion. The proportional data are generated by modeling the corrosion process using mechanistic-empirical models and Monte Carlo simulations. **DOI: 10.1061/(ASCE)BE.1943-5592.0001101.** *© 2017 American Society of Civil Engineers.*

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Introduction

In many existing bridge management systems (BMS) the deterioration of reinforced concrete (RC) bridge elements is modeled using Markov models. Markov models are stochastic models that define the space of the physical condition of the elements into a range of discrete condition states. The transition probabilities used in this type of model can be developed with little effort, using the results of time-ordered visual inspections, which are regularly conducted and stored in databases, or will be regularly conducted in the future.

Existing data involving time-ordered detailed changes, however, are often suitable for the estimation of transition probabilities. There are numerous reasons for this including (1) lack of data because past inspection results were not archived, (2) inconsistent data in time due to inconsistently conducted monitoring programs, (3) biased data due to the lack of strict guidelines for the inspector, and (4) data with measurement errors due to faulty measurement equipment. If existing data cannot be used, one often relies on expert opinion. The poor estimation of transition probabilities, however, affects the prediction of the future condition of bridge elements, which in turn can affect the determination of the optimal intervention strategies for the elements and, therefore, the development of the intervention programs to be implemented.

Despite progressive development in the field of monitoring over the last decades so that data can be collected more frequently and more accurately, the previously described four reasons are still pertinent problems widely seen in practice. They are especially pertinent in developing nations, in which there is too little attention paid to managing infrastructures, including a vast number of concrete bridges. Many concrete infrastructures built in the last decades are in poor condition due to a lack of regular maintenance and retrofitting, and monitoring. Infrastructure managers in those situations are making strategic decisions purely based on mechanistic-empirical models, whose parameter values can be determined at a single time when required.

To overcome this shortcoming, a methodology has been developed to estimate the transition probabilities based on aggregated and proportional sample data. Proportional sample data is data in which only the proportion of the elements analyzed in each state at discrete times are known. There is no information on the time path behavior of each individual element.

The methodology estimates transition probabilities using proportional sample data generated using mechanistic-empirical models. The term *mechanistic-empirical* here refers to a combination of a mechanistic model, which is based on physical behaviors of the elements, and an empirical model, which is based on direct observations, measurements, and extensive data records. The model is used to develop the transition probabilities for the RC bridge element that is affected by chloride-induced corrosion.

The remainder of the paper is organized as follows. In the following section a short background of Markov and mechanistic-empirical models is given to help situate the reader. The methodology, with a focus on the *restricted least-squares transition probability estimator*, is presented next. Afterward, an example is given using the methodology. Finally, a summary of the work and suggestions for future work in this area are given.

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Background

Markov Models

The Markov model discussed can be characterized by the following:

1. There are a finite number of possible outcomes (e.g., condition states) i (i = 1,2,…,I), which are discrete random variables X_t (t = 0,1,2,…,T), taking a finite number of equidistant time points t in a sample space.
2. The probability distribution of an outcome of a given point in time depends only on the outcome of the previous point in time

$$\mathbb{P}(X_t = x_t | X_{t-1} = x_{t-1}, X_{t-2} = x_{t-2}, \dots) = \mathbb{P}(X_t = x_t | X_{t-1} = x_{t-1}) \quad \forall t \tag{1}$$

where $\mathbb{P}(X_t = x_t | \dots)$ denotes the conditional probability density function for X_t.

1. The assumption that if $X_{t-1} = i$ and $X_t = j$, then

$$\mathbb{P}(X_t = j | X_{t-1} = i) = \pi_{ij}(t) = \pi_{ij} \quad \forall t \tag{2}$$

where π_{ij} = constant transition probability associated with a change from state i to state j. These transition probabilities π_{ij} can be arranged as a transition probability matrix Π, reflecting every pair of states i, j (i,j = 1,2,…,I).

When inspection data have been collected at constant time intervals, the estimation of the transition probabilities can be a straightforward process, e.g., the division of the number of transitions between states by the total number of states. However, when data have been collected at nonconstant time intervals, a statistical approach, such as the likelihood estimation approach or Bayesian approach, can be used (Tsuda et al. 2006; Kobayashi et al. 2012; Lethanh et al. 2015b).

When data have not been collected in sufficient quality or quantity to accurately estimate the transition probabilities, they are often estimated, at least in practice, using expert opinion. When using the unreliable transition probabilities, errors in the estimation of the condition evolution of the elements are likely to incur. Because the condition evolution of elements is important in the estimation of intervention strategies and intervention programs (Walbridge et al. 2013; Fernando et al. 2015; Lethanh et al. 2015a), using unreliable transition probabilities might lead to under- and overestimation of necessary budgets, and this problem should be avoided. Under those circumstances, determining reliable transition probabilities from mechanistic-empirical models offers a way to improve these estimates.

Mechanistic-Empirical Models

Mechanistic-empirical models are used extensively to estimate the deterioration of engineering elements. Example models for concrete elements are those developed by DuraCrete (1999, 2000a, b), VTT Technical Research Centre of Finland (2003), *fib* (2006), or Life-365 Consortium III (2013). These models are usually developed with a solid understanding of the physical processes at work in the RC elements, insights provided by laboratory or in situ tests, and data collected to evaluate the actual state of the element.

Mathematically, mechanistic-empirical models are defined as functions of sets of parameters. For example, a model of the deterioration of an RC element may include the level of chloride C_{cl}, initial level of chloride ion concentration C_s, the distance x_{cl} from the concrete surface, and chloride diffusion coefficient D_{cl} (Kirkpatrick et al. 2002)

$$C_{\mathrm{cl}}(x_{\mathrm{cl}}, t) = C_s\left[1 - \mathrm{erf}\left(\frac{x_{\mathrm{cl}}}{2\sqrt{D_{\mathrm{cl}} \cdot t}}\right)\right] \tag{3}$$

where erf[·] = error function.

When the values of parameters are determined, the continuous evolution of a deterioration indicator (e.g., level of chloride) over time can be estimated deterministically. To include the uncertainty in prediction, probabilistic distributions (e.g., normal distribution, lognormal distribution) are normally used to represent the parameters of models, and Monte Carlo simulations are run using different values of the parameters drawn from these distributions.

Transition Probabilities Based on Mechanistic-Empirical Models

When mechanistic-empirical models are used to estimate transition probabilities, a decision has to be made about how to determine if there is indeed a good fit with deterioration predicted using the resulting Markov model and the deterioration predicted using the mechanistic-empirical model. One of the first works in the area is that of Roelfstra et al. (1999, 2004) who assumed that each element could transition no more than one condition state per unit of time and obtained a best fit using the least-squares estimation approach. The best fit was determined for conditional vectors at 100, 150, and 200 years. The validity of the assumption of not transitioning more than one state in one unit of time depends on the speed of deterioration, but it is an assumption made by many researchers (Jiang et al. 1988; Al-Subhi et al. 1989; Mishalani and Madanat 2002; Robelin and Madanat 2007) and used in the past for the estimation of transition probabilities for use in BMS, such as *Pontis 4.1* (Golabi and Shepard 1997; Soderqvist and Veijola 1998; Thompson et al. 1998, 1999; FHWA 2002) and *Kuba-MS 5.1* (Hajdin 2003, 2006). However, obtaining the transition probabilities using only one single point in time to minimize the errors led to a divergence between the deterioration curves obtained with Markov chains and the ones obtained by mechanistic-empirical modeling.

Another way to establish a best fit between the models is to minimize the errors between the probability of being in each state in each time interval in the investigated period predicted using the Markov model and the mechanistic-empirical model, instead of just in one period of time. Although computationally more intensive, this could yield more detailed and better approximations of the future behavior of the elements. This can be done by first estimating the probable values of the deterioration indicators, such as level of chloride concentration or the crack width, in each time interval throughout the investigated time period. These can then be summarized as a state vector, i.e., at time t there is a probability of being in each state i, which is referred to as proportional data (Lee et al. 1972), and then estimating the transition probabilities using these state vectors.

Although the use of proportional data to estimate transition probabilities has not been used in the field of bridge management, there has been substantial early work in the fields of econometric and financial engineering (Lee et al. 1972). As summarized in Lee et al. (1970), it started in the 1950s when Miller (1952) proposed a discrete Markov process in psychology. He defined a stochastic relation formulation as a basis for describing a linear statistical model for estimating the transition probabilities from proportional data using an unrestricted least-squares transition probability estimator. Later, a restricted least-squares transition probability estimator was developed by Goodman (1953). Since then, there has been a significant amount of research done on improving the estimation of transition probabilities. Many of these have concentrated on the use of the maximum likelihood estimation and Bayesian estimation

J. Bridge Eng., 2017, 22(10): 04017063

approaches using examples from different fields (Lee et al. 1972; Lancaster 1990; Kobayashi et al. 2012).

Methodology

The tasks used to estimate the optimal transition probabilities based on proportional data generated by mechanistic-empirical models include the following: (1) select or develop mechanistic-empirical models, (2) estimate parameter values, (3) define condition states, (4) generate values of condition indicators over time, (5) aggregate probable condition states in each time interval, (6) estimate transition probabilities, and (7) evaluate results and the value of error terms.

This methodology is illustrated in Fig. 1 and explained in more detail in the following sections. Particular attention is given to Task 6, the estimation of the transition probabilities.

Task 1: Select or Develop Mechanistic-Empirical Models

The deterioration process of concrete elements due to reinforcement corrosion is commonly described in two phases, initiation and propagation (Tuutti 1982).

1. During the initiation phase the RC structure is exposed to environmental and mechanical effects. The penetration of chloride ions and carbon dioxide into the concrete can lead to steel corrosion when the penetration front reaches the critical depths considering the embedded reinforcement. If the onset of corrosion has occurred, the initial phase ends and the propagation phase starts.
2. During the propagation phase the process of corrosion proceeds, which leads to a reduction in the reinforcement cross section and the accumulation of corrosion products (*rust*). The reduction of the cross section affects the capacity of the RC structure, which may lead to structural failure. The expanded volume of corrosion products may cause cracking and spalling of the covering concrete.

Several mechanistic-empirical models have to be used to describe the complex physical phenomena of chloride-induced corrosion. For the methodology presented in this paper, current state-of-the-art models for the concrete deterioration are used, considering the following elements:

- A probabilistic model for chloride penetration based on Fick's second law of diffusion [Eq. (3)]
- A simplified model for corrosion propagation, based on cross-section loss

$$P(t) = \int_0^t V_{\text{corr}} \cdot \alpha \cdot w_t \cdot t \cdot dt \tag{4}$$

where V_{corr} = corrosion rate coefficient (millimeters per year); w_t = wet period in a year (i.e., the fraction of the year that corrosion is active); and α = pitting factor that takes nonuniform corrosion of the reinforcement bars into consideration.

- A crack model, which takes the section loss into account

$$w(t) = w_0 + \beta\,[P(t) - P_0] \tag{5}$$

where $w(t)$ = crack width (millimeters) over time; β = parameter that controls the propagation; w_0 = crack width when it is visible ($\approx$ 0.05 mm); P_0 = amount of loss of reinforcement bar diameter (millimeters) when crack width is visible; and $P(t)$ = amount of loss of the reinforcement bar diameter (millimeters) at time t.

Task 2: Estimate Parameter Values

The deterioration process of concrete is closely related to the environment. For example, the risk of chloride-induced corrosion is higher in coastal environments than in the interior of the country. In mechanistic-empirical models (environmental) parameters are used as an initial condition for the deterioration models. In this context, the user of the model has to decide which assumption is accurate for the present situation, e.g., if the element is directly exposed to rain, the concrete can have a high level of relative humidity. Measurements are helpful here in reducing uncertainty. Service life models, such as DuraCrete (2000b), provide valuable information about the durability characteristics of concrete structures and environmental parameters.

Task 3: Define Condition States

In existing BMS, the condition states are described both qualitatively and quantitatively and illustrated by images and drawings. Unfortunately, there is little connection from these condition states and the structural behavior of the elements. One of the purposes of BMS, however, is to plan and suggest interventions to execute before any significant impairment of the bridge occurs. In this sense, the purpose of a condition assessment is to serve as a basis for planning of interventions, and the condition states should fit these needs (Roelfstra et al. 2004). This means that condition states are found keeping in mind the purpose of the BMS, and the conditions predicted using mechanistic-empirical models need to be mapped to them.

Task 4: Generate Values of Condition Indicators over Time

Based on the mechanistic-empirical models [Eqs. (3–5)] and their parameter values, condition indicators can be randomly generated by Monte Carlo simulations. The parameter values need to be set taking into consideration the environment of the element. For example, if the concrete cover depth is measured with a thickness of 25 mm and the exposure is observed to be a splash zone, the deterioration rate of the considered element can be adjusted to these input parameters.

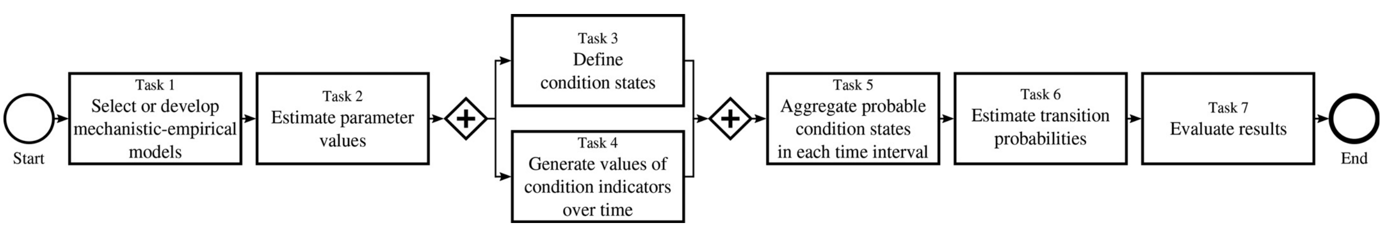

Fig. 1. Methodology

J. Bridge Eng., 2017, 22(10): 04017063

Task 5: Aggregate Probable Condition States in Each Time Interval

After sampling, the values of condition indicators form the mechanistic-empirical models aggregated to obtain the proportions of each condition state i for each time period $t-1$ to t. These proportions, referred to as *observed proportions* $\xi_j(t)$ ($j = 1,2,\ldots,I$), are used in the next task to estimate the transition probabilities.

Task 6: Estimate Transition Probabilities

Assuming that only aggregated sample data (e.g., from mechanistic-empirical models) are available, the transition probabilities can be estimated using the previous notation as follows:

$$\mathbb{P}(X_{t-1}=i, X_t=j) = \mathbb{P}(X_{t-1}=i)\mathbb{P}(X_t=j|X_{t-1}=i) \tag{6}$$

respectively

$$\mathbb{P}(X_t=j) = \sum_i \mathbb{P}(X_{t-1}=i)\mathbb{P}(X_t=j|X_{t-1}=i) \tag{7}$$

or

$$\tau_j(t) = \sum_i \tau_i(t-1)\pi_{ij} \tag{8}$$

where $\tau_j(t)$ and $\tau_i(t-1)$ = true probabilities, which cannot be observed due to a lack of data. By replacing the true values in Eq. (8) with the observed proportions $\xi_j(t)$ and $\xi_j(t-1)$, the following stochastic relationship can be observed:

$$\xi_j(t) = \sum_i \xi_i(t-1)\pi_{ij} + \varepsilon_j(t) \tag{9}$$

where $\varepsilon_j(t) = \xi_j(t) - \tau_j(t)$ is an error term, which accounts for the difference between the actual and estimated occurrence of $\xi_j(t)$. By minimizing the error term $\varepsilon_j(t) \to 0$, the best estimate of the actual probabilities can be found.

Following the work of Lee et al. (1972), Eq. (9) can be rewritten in a standard matrix representation:

$$\xi = \Xi\pi + \varepsilon \tag{10}$$

or

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_I \end{bmatrix} = \begin{bmatrix} \Xi_1 & 0 & \cdots & 0 \\ 0 & \Xi_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Xi_I \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_I \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_I \end{bmatrix} \tag{11}$$

where $\xi_j = T \times 1$ vector of sample points; $\pi_j = [\pi_{1j}, \pi_{2j}, \ldots, \pi_{Ij},] = I \times 1$ vector of unknown transition parameters to be estimated; $\varepsilon_j = T \times 1$ error vector with $\mathbb{E}[\varepsilon_j] = 0$; and $\Xi_j = T \times I$ matrix with

$$\Xi_j = \begin{bmatrix} \xi_1(0) & \xi_2(0) & \cdots & \xi_I(0) \\ \vdots & \vdots & & \vdots \\ \xi_1(t-1) & \xi_2(t-1) & \cdots & \xi_I(t-1) \\ \vdots & \vdots & & \vdots \\ \xi_1(T-1) & \xi_2(T-1) & \cdots & \xi_I(T-1) \end{bmatrix} \tag{12}$$

Using the conventional least-squares estimator as a basis for obtaining the estimates of the transition probabilities yields the following minimization problem:

$$\widehat{\beta} = \varepsilon^T\varepsilon = (\xi - \Xi\pi)^T(\xi - \Xi\pi) \tag{13}$$

subjected to

$$\sum_j \pi_{ij} = 1 \qquad \forall i \tag{14}$$

$$\pi \geq 0 \tag{15}$$

$$\pi_{ij} = 0 \qquad \forall i > j \tag{16}$$

Eq. (14) represents the row sum condition. To ensure that the probabilities per condition state sum up to 1, the nonnegative condition in Eq. (15) allows only positive values, and Eq. (16) ensures that the condition states can only get worse, and not better.

Task 7: Evaluate Results

Finally, obtained transition probabilities are examined to ensure that they are optimal. This can be done using the total sum of error terms for all condition states, and for each condition state at any particular point in time. Along with the error term check, visual inspection on duration of time staying in each condition state and distribution of each condition state over time can also be performed. Ideally, it is expected that the set of transition probabilities resulting in the minimum value of the total sum of error terms compared with that of other sets is the optimal one.

Example

The methodology is demonstrated by using it to determine the transition probabilities of an RC bridge deck exposed to chloride-induced corrosion. It is assumed that the chloride concentration and the crack width are the essential condition indicators.

Task 1: Select or Develop Mechanistic-Empirical Models

The current state-of-the-art models for the concrete deterioration were used, and the initiation and propagation models proposed by DuraCrete (2000b) were used.

To model the initial phase of the corrosion process, Eq. (3) was used as a base. This mechanistic-empirical model has the advantage that parameters, such as the diffusion coefficient, can be expanded as functions of other parameters. This allows one to approximate the environmental condition of the element in a more realistic manner. According to DuraCrete (2000b) D_{cl} can be defined as

$$D_{cl} = D_{cl}(t_0) \cdot \left(\frac{t_0}{t}\right)^n = k_e \cdot k_t \cdot k_c \cdot D_0 \cdot \left(\frac{t_0}{t}\right)^n \tag{17}$$

where k_e = environmental parameter; k_t = test method parameter; k_c = executions parameter; D_0 = empirical diffusion coefficient; t_0 = reference time; and n = age factor.

To model the propagation phase, the mechanistic-empirical models for the cross-section loss [Eq. (4)] and the crack width [Eq. (5)] were used. Similar to the diffusion coefficient in the initiation phase, the amount of lost cross section when the crack initially occurred (P_0) can be described, taking the environment into account

$$P_0 = a_1 + a_2 \cdot d_c/\phi + a_3 \cdot f_{t,\mathrm{spl}} \tag{18}$$

where a_1, a_2, and a_3 = regression parameters; d_c = concrete cover depth; ϕ = diameter of the reinforcement bar; and $f_{t,\mathrm{spl}}$ = tensile splitting strength.

Task 2: Estimation of Parameter Values

Each parameter of Eqs. (3)–(5), (17), and (18) can be expressed as a random variable depending on the location (e.g., located in coastal areas or not); environment, such as the exposure zone (e.g., submerged, tidal, splash, for a coastal location and dry or wet for an inland location); the curing time; the water-cement ratio; and the concrete type of the element.

For illustration purposes the following assumptions about the element were made:

- The bridge was located in Switzerland (inland).
- A portland cement with a water-cement ratio of 0.4 was used.
- The curing time was 7 days.
- The element was located in a wet environment, i.e., parts of the element could be exposed to rain.
- Concrete cover depths (d_c) of 15, 25, and 35 mm were evaluated.

Table 1. Input Parameters for the Initiation Phase

Parameter	Distribution	μ	σ	Table[a]
D_0	Normal	220.752	25.41	8.2
C_s	Lognormal	0.24	0.16	8.6
k_e	Gamma	0.265	0.045	8.4
k_t	Normal	0.832	0.024	8.8
k_c	Deterministic	1	—	8.7
t_0	Deterministic	0.0767	—	8.13
n	Beta	0.37	0.07	8.3

[a]Data from DuraCrete (2000b).

Table 2. Input Parameters for the Propagation Phase

Parameter	Distribution	μ	σ	Table[a]
w_0	Normal	0.05	0.005	—
β	Normal	0.0086	0.005	9.16
V_{corr}	Weibull	0.03	0.04	9.3
w_t	Normal	0.75	0.2	9.3
α	Normal	9.28	4.04	9.2
a_1	Normal	74.4	5.7	9.16
a_2	Normal	7.3	0.06	9.16
a_3	Normal	–17.4	3.2	9.16
ϕ	Deterministic	20	—	—
$f_{t,\mathrm{spl}}$	Deterministic	2.6	—	—

[a]Data from DuraCrete (2000b).

The corresponding distributions and parameters are documented in Tables 1 and 2 based on DuraCrete (2000b), in which other side specific parameters are also listed.

Task 3: Define Condition States

Five condition states were defined (Table 3), corresponding to the chemical and physical criteria described by the mechanistic-empirical models. A segment in Condition State 1 was defined as having less than 0.24% by weight chloride ion content, which was equal to the surface concentration. Condition State 2 was defined as having a chloride ion concentration of less than 0.48% by weight at the surface of the reinforcement. This represented a contaminated concrete and the transition from the initiation to the propagation phase, in which 0.48% by weight defines the critical chloride concentration C_{crit} and the depassivation of the steel surface begins. Condition States 3–5 were defined in terms of the crack width. In State 3 corrosion has already started; however, time is necessary for the crack to reach the surface. Cracks on the surface can be visually detected for elements in Condition State 4. An element in Condition State 5 has crack widths larger than 0.5 mm, which indicates that safety may be a problem.

Task 4: Generate Values of Condition Indicators

Based on the chosen mechanistic-empirical models and their parameters, 20,000 simulations of each condition indicators C_{cl} and w in each year in a period of 100 years were performed. A Monte Carlo approach was used to generate the data. It was assumed that the variables were uncorrelated.

Table 4. First 15 Years of Aggregated Data from the Simulation of 20,000 Individuals per Year

t	s_1	s_2	s_3	s_4	s_5
1	1.0000	0.0000	0.0000	0.0000	0.0000
2	1.0000	0.0000	0.0000	0.0000	0.0000
3	0.9999	0.0001	0.0000	0.0000	0.0000
4	0.9980	0.0020	0.0000	0.0000	0.0000
5	0.9901	0.0099	0.0000	0.0000	0.0000
6	0.9748	0.0249	0.0003	0.0000	0.0000
7	0.9518	0.0470	0.0012	0.0000	0.0000
8	0.9185	0.0786	0.0030	0.0000	0.0000
9	0.8801	0.1135	0.0064	0.0000	0.0000
10	0.8357	0.1515	0.0127	0.0001	0.0000
11	0.7908	0.1900	0.0188	0.0003	0.0000
12	0.7463	0.2260	0.0269	0.0007	0.0001
13	0.7032	0.2588	0.0366	0.0013	0.0002
14	0.6627	0.2884	0.0462	0.0023	0.0005
15	0.6243	0.3118	0.0593	0.0038	0.0009

Table 3. Definition of the Condition States

Phase	CS	Description	Indicator	Criteria
Initiation	s_1	New/partial new concrete contaminated	Chloride concentration at the reinforcement bar	$0 \le C_{\mathrm{cl}} \le 0.24$
	s_2			$0.24 \le C_{\mathrm{cl}} \le 0.48$
Propagation	s_3	Corrosion has initiated, no visible cracking	Crack width	$C_{\mathrm{cl}} > 0.48,\ w \le 0.25$
	s_4	Visible cracking has occurred		$0.25 < w \le 0.5$
	s_5	Visible cracking has occurred and cover has spalled		$w > 5$

Note: CS = condition states.

Task 5: Aggregate Probable Condition States in Each Time Interval

To obtain the set of proportional data, the values obtained from the simulations were used to classify the simulation as having generated a discrete condition state. To calculate the observed proportions at any time t, the values of chloride concentration C_{cl} and the crack width w were evaluated according to Table 3. The amount of cases for each condition state were divided by the total amount of simulations to get a state probability at any time t. A sample of the observed proportions $\xi_i(t)$, for a concrete cover depth of 25 mm, is shown in Table 4. For reason of brevity the complete list for 100 years is not given, but it can be easily obtained by running the code published on the GitHub site.

Table 5. Deterioration Matrices

d_c	CS	s_1	s_2	s_3	s_4	s_5
15	s_1	0.7761	0.2048	0.0191	0.0000	0.0000
	s_2	0.0000	0.8278	0.1667	0.0055	0.0000
	s_3	0.0000	0.0000	0.9688	0.0220	0.0092
	s_4	0.0000	0.0000	0.0000	0.9578	0.0422
	s_5	0.0000	0.0000	0.0000	0.0000	1.0000
25	s_1	0.9282	0.0578	0.0139	0.0001	0.0000
	s_2	0.0000	0.9430	0.0570	0.0000	0.0000
	s_3	0.0000	0.0000	0.9397	0.0494	0.0109
	s_4	0.0000	0.0000	0.0000	0.9209	0.0791
	s_5	0.0000	0.0000	0.0000	0.0000	1.0000
35	s_1	0.9886	0.0085	0.0011	0.0009	0.0009
	s_2	0.0000	0.9901	0.0071	0.0020	0.0008
	s_3	0.0000	0.0000	0.9463	0.0392	0.0145
	s_4	0.0000	0.0000	0.0000	0.9147	0.0853
	s_5	0.0000	0.0000	0.0000	0.0000	1.0000

Task 6: Estimate Transition Probabilities

Making use of the restricted least-squares transition probability estimator [Eq. (13)], the transition probabilities in the deterioration matrix were calculated. Convex optimization was used to minimize the error term. The matrices obtained for the three different concrete cover depths are presented in Table 5.

The program for the restricted least-squares transition probability estimator, used for this example, was coded in Python using the *Python CVXPY 1.0*-embedded modeling language for convex optimization problems (Diamond and Boyd 2016). The source code of the restricted least-squares transition probability estimator (*rlstpe 1.0.0*) is distributed on GitHub under a GNU general public license.

Task 7: Evaluation Results

Fig. 2 shows the evaluated deterioration curves based on the matrices of Table 5. Because the concrete cover depth is an important factor for chloride-induced corrosion, it is not surprising that the deterioration curves show different behavior, depending on the depth d_c. In the case of a small concrete cover depth [Fig. 2(a)], the probability that chloride-induced corrosion and cracks will occur in an early stage is much higher, as in the case of a larger concrete cover depth [Fig. 2(c)].

To evaluate the goodness of fit between the estimated transition probabilities and the simulated values, the error terms for

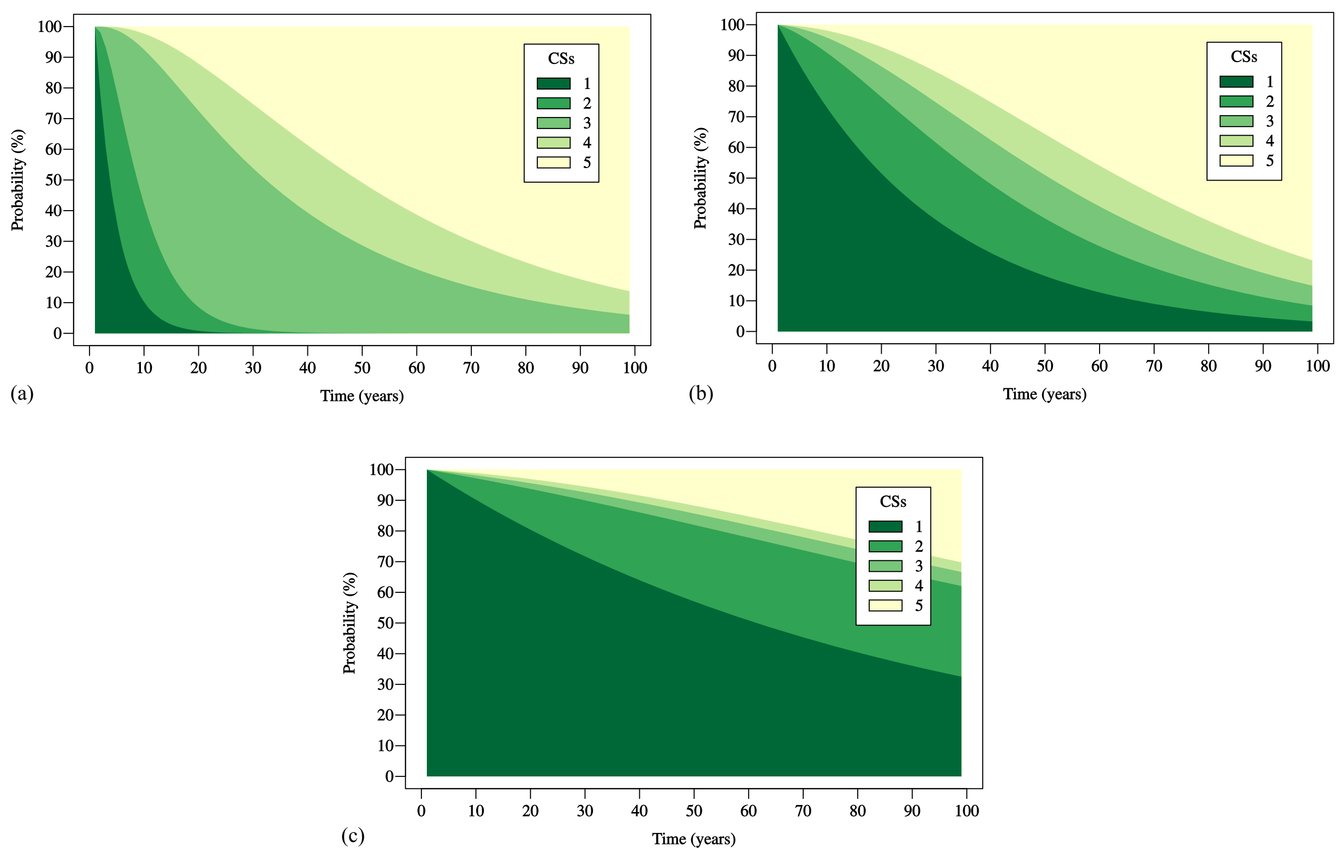

Fig. 2. Evaluated deterioration curves: (a) concrete cover depth $d_c = 15$ mm; (b) concrete cover depth $d_c = 25$ mm; (c) concrete cover depth $d_c = 35$ mm

J. Bridge Eng., 2017, 22(10): 04017063

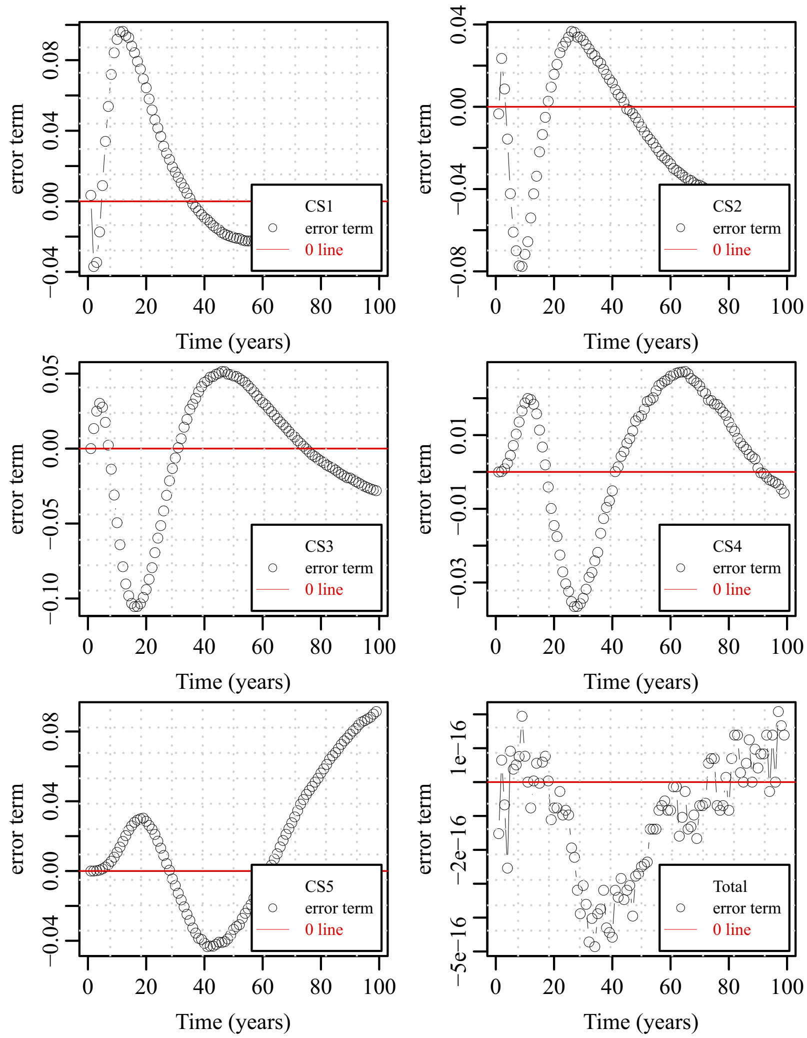

Fig. 3. Value of error terms

each condition state at each time step are plotted in Fig. 3. It can be seen that the distributions of error terms are not stable. This instability is also true for the total error term, which is plotted in the last graph of the figure. This instability is most likely because the proportional data are generated at every time step, but the proportional data of one time step are not dependent on the proportional data of previous steps.

Conclusions

In this paper, a methodology was presented to estimate the transition probabilities for RC elements using proportional data generated from mechanistic-empirical models. The proposed methodology makes use of a restricted least-squares optimization model to determine the optimal transition probabilities, i.e., those probabilities that yield the minimal total sum of error terms between the predicted distribution of the condition state in each time interval over the investigated time period using the Markov model and the mechanistic-empirical models directly.

This methodology offers a way to estimate transition probabilities when there are no, or only insufficient, data available, but the mechanistic-empirical models exist. One example is the case of corrosion, in which no or only sparse inspection data are available, but various mechanistic-empirical models are available. The methodology was demonstrated by using models to estimate the transition probabilities for a simple example of an RC bridge deck exposed to chloride-induced corrosion. It was demonstrated that the condition evolution predicted using the Markov model was close to that predicted by the mechanistic-empirical models. The total error of the estimated transition probabilities was in the range between 1×10^{-10} and 5×10^{-16}.

A weakness of the methodology is that the distributions of condition states in each time interval over the investigated using the mechanistic-empirical model is done as if the each time interval is independent of the ones before it. Future work will be concentrated on improving this, particularly with the use of the Bayesian estimation approach to overcome the limitation. Other future work should focus on how such a methodology could be integrated into existing BMS and in estimating the potential improvements in the determination of optimal intervention strategies and programs when compared with estimating transition probabilities based on expert opinion, which is often done when no data exist.

Notation

The following symbols are used in this paper:

a_i = regression parameter;

C_{cl} = chloride concentration;

C_s = initial (surface) chloride concentration;
D_{cl} = chloride diffusion coefficient;
D_0 = empirical diffusion coefficient;
d_c = concrete cover depth;
$f_{t,spl}$ = tensile splitting strength;
I = total number of condition states;
i = condition state i;
k_c = executions parameter;
k_e = environmental parameter;
k_t = test method parameter;
n = age factor;
P = cross-section loss;
P_0 = cross-section loss when cracks are visible;
T = number of points in time;
t = time;
t_0 = reference time;
V_{corr} = corrosion rate coefficient;
w = crack width;
w_t = wet period in a year;
w_0 = initial visual crack width;
X = random variable;
x = realization of a random variable;
x_{cl} = distance from the concrete surface;
α = pitting factor;
β = propagation parameter;
$\varepsilon_j(t)$ = error term for state j at time t;
Ξ = matrix with I blocks of identical Ξ_j on the diagonal;
Ξ_j = matrix of observed proportions;
$\xi_j(t)$ = observed proportion in state j at time t;
Π = transition probability matrix;
π_{ij} = transition probability;
$\tau_j(t)$ = true proportion for state j at time t; and
ϕ = reinforcement bar diameter.

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